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Suppression of the plasma high-frequency electrical conductivity under a radiation field

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Abstract

From the quantum mechanical viewpoint we derive the dielectric function of an electron plasma system in the presence of a radiation field. By using time-dependent wavefunctions for plasma electrons under the external ac field, we calculate the charge density fluctuation of the electronic system under a weak probing potential and the spectrum of the collective excitation is calculated and found to be strongly dependent upon the amplitude and frequency of the radiation field. We show, in the classical limit, that the reduction of the collective excitation frequency under the radiation field can be associated with the suppression of the plasma high-frequency reactive electrical conductivity. The result is consistent with the recent experimental observation of increased high-frequency mobility in two-dimensional electron gases under a radiation field.

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1. Introduction

Owing to the development of high-power radiation sources, there has been increasing interest in the study of the interaction of intense radiation fields with plasmas. Several phenomena were investigated amongst other, plasma heating via inverse bremsstrahlung regarding radiation fusion experiments [1–11] and plasma wave instabilities [12]. Another interesting aspect concerning the radiation–plasma interaction and the one which pertains to us here is the possibility of a radiation field control of the plasma dielectric properties. As we know, the investigation of the dielectric function of an electron gas has played an important role in modern electronics. The dielectric function measures the strength of a gas of electrons interacting via their long-range force such as Coulomb potential [13, 14]. Therefore, it is of paramount importance to study the elementary electronic excitation from a plasma electron gas. For instance, the dielectric response of a plasma to electromagnetic waves (e.g. higher modes of incident microwaves) and a radio-frequency dielectric response for a Tokamak have been investigated [15, 16]. Numerical experiments were carried out for the discharge in argon at atmospheric pressure and the characteristics of the discharge plasma were found to be dependent upon the applied electromagnetic field mode, power and frequency.

Aside from the experimental focus on plasma heating and confinement, it is of value to examine how an external radiation-field affects such a fundamental quantity as the dielectric function and its consequence on the plasma high-frequency electrical conductivity $\sigma(\omega)$ which is our main motivation in the current study.

Although the examples of applications of our calculation mention plasma conditions where quantum effects seem to be completely negligible (since the Fermi energy is much less than the plasma temperature), we will tackle the problem of the electron gas in a plasma under the radiation field using the quantum mechanical approach instead of the usual classical approach which is simpler. However, in the present case the quantum mechanical approach seems to be more suitable than the classical one because it enables us to perform canonical transformation in order to solve the time-dependent electron motion in the radiation field. As we know, in the absence of nonlinear effects, the classical permittivity of a plasma and the dispersion relation for electrostatic and electromagnetic modes are well known, and the modes are uncoupled. By making use of the quantum mechanical formalism the 'radiation field' effects enter the problem straightforward from the outset through the electron time-dependent Schrödinger equation. The obtained time-dependent electron wavefunction is then used to calculate the electronic state in a local potential which permits us to derive the dielectric response of the system in the classical limit. This kind of approach has been employed to plasma systems as seen elsewhere [13, 14].

2. Formalism

In what follows, we set up the formalism for the dielectric response function of a weakly ionized plasma in the presence of an external electromagnetic field (EM) from which the modes of the collective excitation of the system are determined. Then by making use of the usual relationship between the dielectric and the conductivity functions we provide a relation of the high-frequency electrical conductivity and field-dependent collective excitations.

We assume here the plasma to be the one in which the out of equilibrium electron distribution function corresponding to a small anisotropy and spatial inhomogeneity of electrons deviate only slightly from the equilibrium distribution function (homogeneous and isotropic). As for the external field, if the distance over which the amplitude of field changes is large in comparison with the size of the charges placed in the plasma, the initial Debye screening radius (r_D) and the amplitude of the electron oscillations in the wave field, we can use dipole approximation. Classically, the electromagnetic wave can propagate in a plasma only if its frequency ω is higher than the plasma frequency Ω_{pe}. For $\omega < \Omega_{pe}$ wave penetration into the plasma only occurs provided the length of the sample is much smaller than the penetration depth of the field into the sample, namely $\delta = c\left(\Omega_{pe}^2 - \omega^2\right)^{-1/2}$. Also, for simplicity and convenience we limit our calculation to field amplitudes in the interval of 0.1–10 V cm^{-1} (non-intense field regime) which are far below those EM fields which accelerate the electrons to the relativistic regime. Relativistic corrections only matter for longer wavelength radiation fields at foreseeable intensities as high as 10^{16} W cm^{-2}. Nevertheless, these effects are insignificant for wavelengths smaller than 10 μm [17].

To begin with, let us consider an electron of the plasma under a radiation field. We choose the EM field to be such that its electric field is $E(t) = E\hat{e}_x \cos(\omega t)$, where E and ω are the field amplitude and frequency, respectively. For notational convenience, we use $\hbar = c = V = 1$, with c being the speed of light and V the normalization volume. In the absence of a radiation field, the Schrödinger equation for a single plasma electron is given by

$$\frac{\vec{p}^2}{2m_e}\psi(\vec{r}, t) = \mathrm{i}\frac{\partial\psi(\vec{r}, t)}{\partial t}, \tag{1}$$

and the wavefunction of the electron is simply a plane wave $\psi(\vec{r}, t) = \mathrm{e}^{\mathrm{i}\vec{k}\cdot\vec{r}}\mathrm{e}^{-\mathrm{i}\varepsilon_{\vec{k}}t}$, where $\varepsilon_{\vec{k}} = \vec{k}^2/2m_e$ and m_e is the electron mass.

On the other hand, in the presence of an intense radiation field the electrons are strongly coupled to the electromagnetic field. So the vector potential is given in the form

$$\vec{A}(t) = \frac{E}{\omega}\hat{e}_x \sin(\omega t). \tag{2}$$

In this case, the time-dependent Schrödinger equation is given by

$$\mathrm{i}\frac{\partial\Psi(\vec{r}, t)}{\partial t} = H\Psi(\vec{r}, t), \tag{3}$$

where $H = (1/2m_e)[\vec{p} - e\vec{A}(t)]^2$ and e is the elementary charge. It can be shown that (1) and (3) are related to each other by a simple unitary transformation [6–8]

$$U\left[\mathrm{i}\frac{\partial}{\partial t} - \frac{\vec{p}^2}{2m_e}\right]U^{\dagger} = \mathrm{i}\frac{\partial}{\partial t} - \frac{1}{2m_e}[\vec{p} - e\vec{A}(t)]^2 \tag{4}$$

where

$$U = \exp(-\mathrm{i}2\gamma_1\omega t)\exp\{\mathrm{i}\gamma_0 k_x[1 - \cos(\omega t)]\}\exp[\mathrm{i}\gamma_1 \sin(2\omega t)] \tag{5}$$

and the wavefunction can be written as

$$\Psi_{\vec{k}}(\vec{r}, t) = U\mathrm{e}^{-\mathrm{i}\varepsilon_{\vec{k}}t}\,\mathrm{e}^{\mathrm{i}\vec{k}\cdot\vec{r}}, \tag{6}$$

where $\gamma_0 = eE/m_e\omega^2$ and $\gamma_1 = e^2E^2/8m_e\omega^3$. We now employ this time-dependent wavefunction to calculate the electronic state in a local potential (to be determined self-consistently) and to derive the dielectric properties of system. The zeroth-order wavefunction is given by

$$\Psi^0_{\vec{k}} = \exp[\mathrm{i}F(\omega, t)]\exp\{\mathrm{i}\gamma_0 k_x[1 - \cos(\omega t)]\}\,\mathrm{e}^{\mathrm{i}\vec{k}\cdot\vec{r}}\,\mathrm{e}^{\mathrm{i}\varepsilon_{\vec{k}}t}. \tag{7}$$

Here

$$F(\omega, t) = 2\gamma_1\omega t + \gamma_1 \sin(2\omega t) \tag{8}$$

and (7) forms an orthonormal set, $\langle\Psi^0_{\vec{k}}|\Psi^0_{\vec{k}'}\rangle = \delta_{\vec{k},\vec{k}'}$.

In this case, there is no charge fluctuation even in the presence of the radiation field, i.e.

$$\rho^{(0)}_{\vec{k}} = -e|\Psi^0_{\vec{k}}|^2 = -e. \tag{9}$$

The wavefunction of an electron under a local potential can be expanded using the above orthonormal set as

$$\Psi(\vec{r}, t) = \sum_{\vec{k}} a_{\vec{k}}(t)\exp[\mathrm{i}F(\omega, t)]\exp\{\mathrm{i}\gamma_0 k_x[1 - \cos(\omega t)]\}\,\mathrm{e}^{\mathrm{i}\vec{k}\cdot\vec{r}}\,\mathrm{e}^{\mathrm{i}\varepsilon_{\vec{k}}t}, \tag{10}$$

where the coefficient $a_{\vec{k}}(t)$ will be determined through the time-dependent perturbation method.

We now consider a local potential $\phi(\vec{r}, t)$ which can be written as

$$\phi(\vec{r}, t) = \int \mathrm{d}\vec{q} \int \mathrm{d}\Omega\, \mathrm{e}^{\mathrm{i}\vec{q}\cdot\vec{r}} \mathrm{e}^{\mathrm{i}\Omega t} \phi(\vec{q}, \Omega) + \text{c.c.}, \tag{11}$$

where c.c. denotes the complex conjugate of the preceding term. We assume that the local potential is weak and use the time-dependent perturbation [18] to calculate the change of electronic state. The time-dependent Schrödinger equation is now given by

$$\mathrm{i}\frac{\partial \Psi}{\partial t} = (H - e\phi)\,\psi, \tag{12}$$

where H is given by (3). Upon using (10), we obtain the first-order equation

$$\mathrm{i}\frac{\partial a_{\vec{k}'}}{\partial t} = -e \exp\{\mathrm{i}\gamma_0(k_x - k_x')[1 - \cos(\omega t)]\}\, \mathrm{e}^{-\mathrm{i}(\varepsilon_{\vec{k}'} - \varepsilon_{\vec{k}})t} \int \mathrm{d}\vec{r}\, \mathrm{e}^{-\mathrm{i}\vec{k}'\cdot\vec{r}} \phi(\vec{r}, t)\, \mathrm{e}^{\mathrm{i}\vec{k}\cdot\vec{r}}. \tag{13}$$

Substituting the Fourier expansion given by (11) and making use of the generating function of the Bessel function, namely

$$\exp(\mathrm{i}\alpha \cos x) = \sum_m \mathrm{i}^m J_m(\alpha)\, \mathrm{e}^{\mathrm{i}mx}, \tag{14}$$

we obtain

$$a_{\vec{k}+\vec{q}}(t) = (\mathrm{i}e)\, \mathrm{e}^{-\mathrm{i}\gamma_0 q_x} \sum_{m,\Omega} \mathrm{i}^m J_m(q_x\gamma_0)\phi(\vec{q}, \Omega) \int_{-\infty}^{t} \mathrm{d}t' \exp[-\mathrm{i}(\varepsilon_{\vec{k}+\vec{q}} - \varepsilon_{\vec{k}} - \Omega - m\omega)t']. \tag{15}$$

The wavefunction up to first order is now given as

$$\Psi_{\vec{k}}(\vec{r}, t) = \Psi^0_{\vec{k}}(\vec{r}, t) + \sum_{\vec{q}} a_{\vec{k}+\vec{q}}(t)\Psi^0_{\vec{k}+\vec{q}}(\vec{r}, t). \tag{16}$$

Hence, the fluctuation of the charge distribution, namely

$$\rho_{\vec{k}}(\vec{r}, t) = -e[\Psi^*_{\vec{k}}(\vec{r}, t)\Psi_{\vec{k}}(\vec{r}, t) - 1], \tag{17}$$

can be calculated. Neglecting high-order terms in ϕ, we obtain

$$\begin{aligned}\rho_{\vec{k}}(\vec{r}, t) = -e^2 \sum_{\vec{q},\Omega} \sum_m \mathrm{i}^m \phi(\vec{q}, \Omega) J_m(q_x\gamma_0) \\ \times \Bigg\{ \frac{\exp[-\mathrm{i}\gamma_0 q_x \cos(\omega t)] \exp[-\mathrm{i}(\Omega + m\omega)t]}{\varepsilon_{\vec{k}+\vec{q}} - \varepsilon_{\vec{k}} - \Omega - m\omega - \mathrm{i}\eta}\, \mathrm{e}^{\mathrm{i}\vec{q}\cdot\vec{r}} \\ + \frac{(-1)^m \exp[\mathrm{i}\gamma_0 q_x \cos(\omega t)] \exp[\mathrm{i}(\Omega + m\omega)t]}{\varepsilon_{\vec{k}+\vec{q}} - \varepsilon_{\vec{k}} - \Omega - m\omega - \mathrm{i}\eta}\, \mathrm{e}^{-\mathrm{i}\vec{q}\cdot\vec{r}} \Bigg\}.\end{aligned} \tag{18}$$

The contribution to the induced charge density due to the complex conjugate part of the local potential can be calculated with the same method. After some rearrangements we obtain

$$\begin{aligned}\rho_{\vec{k}}(\vec{r}, t) = -e^2 \sum_{\vec{q},\Omega} \mathrm{e}^{\mathrm{i}\vec{q}\cdot\vec{r}} \phi(\vec{q}, \Omega) \exp[-\mathrm{i}\gamma_0 q_x \cos(\omega t)]\, \mathrm{e}^{-\mathrm{i}\Omega t} \sum_m \mathrm{i}^m J_m(q_x\gamma_0)\, \mathrm{e}^{-\mathrm{i}m\omega t} \\ \times \left(\frac{1}{\varepsilon_{\vec{k}+\vec{q}} - \varepsilon_{\vec{k}} - \Omega - m\omega - \mathrm{i}\eta} + \frac{1}{\varepsilon_{\vec{k}+\vec{q}} - \varepsilon_{\vec{k}} + \Omega + m\omega + \mathrm{i}\eta} \right) + \text{c.c..}\end{aligned} \tag{19}$$

And the total density fluctuation of the system can be written as

$$\rho(\vec{r}, t) = \sum_{\vec{k}} f_{\vec{k}} \rho_{\vec{k}}(\vec{r}, t), \tag{20}$$

where

$$f_{\vec{k}}(T, E_\gamma) = n_0 \left(\frac{1}{2\pi m_e k_B T}\right)^{3/2} \exp\left(-\frac{\vec{k}^2}{2m_e k_B T} - \frac{E_\gamma}{k_B T}\right) \tag{21}$$

is the Maxwellian distribution function [19]. Here $E_\gamma = 2\gamma_1\omega$ is the energy of the radiation field, n_0 is the electron gas density, k_B is the Boltzmann constant and T is the plasma temperature. The assumption of the Maxwellian distribution function $f_{\vec{k}}(T, E_\gamma)$ is consistent with the fact that provided the electrons kick frequently other electrons in order to be in thermal equilibrium, they also get energy from the external EM source. On this we would like to strengthen our argument concerning the distribution function as follows. In the presence of a radiation field the electron velocity is shifted by a factor $\vec{v} - \vec{v}_E \sin(\omega t)$, where $\vec{v}_E = e\vec{E}/m_e\omega$, and the electron distribution function is given by

$$f(\omega, E) = n_0 \left(\frac{1}{2\pi}\right)^{3/2} {v_{th}}^3 \exp\left[-\frac{1}{2v_{th}^2}(\vec{v} - \vec{v}_E \sin\omega t)^2\right], \tag{22}$$

with $v_{th} = \sqrt{k_B T/m_e}$ being the thermal velocity. Since we consider the high-frequency regime of EM field for which $\omega t > 1$ (t is the electron relaxation time) the term in $f(\omega, E)$ involves a factor $\exp(\sin\omega t)$ (after the binomial form has been developed). This factor can be transformed into a series of Bessel functions of order s ($s = 0, 1, 2, \ldots$). For $\omega t > 1$ it oscillates very rapidly in time and the only term that survives in the series is the $s = 0$ term thereby resulting in the above expression for $f_{\vec{k}}(T, E_\gamma)$. Hence, (21) is to be interpreted as a time-averaged Maxwellian distribution function (averaged over a period of oscillation of EM field).

Proceeding further, after substitution of (19) into the total density fluctuation expression we obtain

$$\rho(\vec{r}, t) = e^2 \sum_{\vec{q},\Omega} \frac{e^{i\vec{q}\cdot\vec{r}}}{e^{i\Omega t}} \phi(\vec{q}, \Omega) \exp[-i\gamma_0 q_x \cos(\omega t)] \sum_m i^m J_m(q_x\gamma_0)\, e^{-im\omega t}\, \Pi(\vec{q}, \Omega + m\omega) \tag{23}$$

where

$$\Pi(\vec{q}, \Omega) = \sum_{\vec{k}} \frac{f_{\vec{k}+\vec{q}} - f_{\vec{k}}}{\varepsilon_{\vec{k}+\vec{q}} - \varepsilon_{\vec{k}} - \Omega - i\eta} \tag{24}$$

is the usual electron polarizability.

After decomposing the time-dependent factor $\exp[-i\gamma_0 q_x \cos(\omega t)]$ into successive harmonics, which is obtained by substituting

$$\exp[-i\gamma_0 q_x \cos(\omega t)] = \sum_{m'} i^{-m'} J_{m'}(q_x\gamma_0)\, e^{im'\omega t}, \tag{25}$$

the electron density fluctuation can be written as

$$\rho(\vec{r}, t) = -e^2 \sum_{\vec{q},\Omega} \frac{e^{i\vec{q}\cdot\vec{r}}}{e^{i\Omega t}} \phi(\vec{q}, \Omega) \sum_{m,m'} i^{m-m'} J_m(q_x\gamma_0) J_{m'}(q_x\gamma_0)\, e^{-i(m-m')\omega t}\, \Pi(\vec{q}, \Omega + m\omega). \tag{26}$$

Proceeding further from Poisson equation, the induced potential can be calculated from the density fluctuation,

$$\nabla^2\phi_{ind}(\vec{r}, t) = -4\pi\rho(\vec{r}, t). \tag{27}$$

Performing the Fourier expansion for the induced potential we obtain

$$\nabla^2\phi_{\rm ind}(\vec{r},t) = -\sum_{\vec{q},\Omega} q^2\,{\rm e}^{{\rm i}\vec{q}\cdot\vec{r}}\,{\rm e}^{{\rm i}\Omega t}\phi_{\rm ind}(\vec{q},\Omega) + {\rm c.c.}. \tag{28}$$

Combining now (26)–(28), we obtain the Fourier component of the induced potential

$$\phi_{\rm ind}(\vec{q},\Omega) = \frac{4\pi e^2}{q^2}\phi(\vec{q},\Omega)\sum_m J_m^2(q_x\gamma_0)\Pi(\vec{q},\Omega+m\omega). \tag{29}$$

In order to obtain (29) we assumed usual radiation frequencies such that $\omega t > 1$ (here t is to be interpreted as the time of flight of the electron in the plasma under the radiation electric field before colliding with any scattering center), in this case the factor $\exp[-{\rm i}\left(m-m'\right)\omega t]$ oscillates very rapidly and in a period of the radiation field it is vanishingly small. Therefore, the terms for which $m \neq m'$ do not contribute to the induced potential. Hence, (29) is the result for the induced potential, at a steady state, after averaging t in $\rho(\vec{r},t)$ over a period of the radiation field. It tells us that in the presence of the radiation field the potential becomes anisotropic through the Bessel function.

The local potential is given by the sum of the external and the induced potentials, i.e.

$$\phi(\vec{q},\Omega) = \phi_{\rm ext}(\vec{q},\Omega) + \phi_{\rm ind}(\vec{q},\Omega), \tag{30}$$

with the dielectric function given by

$$\phi(\vec{q},\Omega) = \phi_{\rm ext}(\vec{q},\Omega)/\epsilon(\vec{q},\Omega), \tag{31}$$

which leads to

$$\epsilon(\vec{q},\Omega) = 1 - \frac{4\pi e^2}{q^2}\sum_m J_m^2(q_x\gamma_0)\sum_{\vec{k}}\frac{f_{\vec{k}+\vec{q}} - f_{\vec{k}}}{\varepsilon_{\vec{k}+\vec{q}} - \varepsilon_{\vec{k}} - \Omega - m\omega - {\rm i}\eta}. \tag{32}$$

It follows immediately from (32) that in the absence of the radiation field, $\gamma_0 = 0$ and $J_m^2(\gamma_0 q_x) = \delta_{0,m}$ so that

$$\epsilon(\vec{q},\Omega) = 1 - \frac{4\pi e^2}{q^2}\sum_{\vec{k}}\frac{f_{\vec{k}+\vec{q}} - f_{\vec{k}}}{\varepsilon_{\vec{k}+\vec{q}} - \varepsilon_{\vec{k}} - \Omega - {\rm i}\eta}, \tag{33}$$

which is the well-known result for the field-free dielectric function. The dielectric constant derived above is valid for any strength of the radiation field at any electron densities and temperature, provided that the probing potential $\phi_{\rm ext}(\vec{r},t)$ (and the resulting local potential) is weak.

The modes of the collective excitation of the system are determined by the solution of

$$\epsilon(\vec{q},\Omega) = 0. \tag{34}$$

However, the correct units in terms of $\hbar$, c, V must be recovered first. By writing

$$f_{\vec{k}} = \tilde{f}_{\vec{k}}\exp\left(-E_\gamma/k_{\rm B}T\right) \tag{35}$$

where

$$\tilde{f}_{\vec{k}} = n_0\left(\frac{\hbar^2}{2\pi m_{\rm e}k_{\rm B}T}\right)^{3/2}\exp\left(-\frac{\hbar^2\vec{k}^2}{2m_{\rm e}k_{\rm B}T}\right) \tag{36}$$

and taking the classical limit ($\hbar \to 0$) such that $m_{\rm e}\vec{v} = \hbar\vec{k}$, equation (32) becomes

$$\epsilon(\vec{q},\Omega) = 1 - \frac{4\pi e^2}{m_{\rm e}q^2}\sum_m J_m^2(q_x\gamma_0)\int {\rm d}^3v\,\frac{\vec{q}\cdot\frac{\partial\tilde{f}}{\partial\vec{v}}}{\vec{v}\cdot\vec{q} - \Omega - m\omega - {\rm i}\eta}\exp\left(-\frac{E_\gamma}{k_{\rm B}T}\right). \tag{37}$$

Table 1. Parameters for some typical plasmas.

Type	n_0 (m^{-3})	$k_B T$ (J)	Ω_{pe} (s^{-1})
Gas discharge	10^{20}	1.6×10^{-19}	6×10^{11}
Hot plasma	10^{20}	1.6×10^{-17}	6×10^{11}
Hot and diffuse	10^{18}	1.6×10^{-17}	6×10^{10}
Warm plasma	10^{20}	1.6×10^{-18}	6×10^{11}

Since we are mainly concerned with the effects of the EM field amplitude on the dielectric function which comes into it via the Bessel function, we make $m = 0$ in (37). Also, we consider the fact that the phase velocity of the wave is much greater than the thermal velocities and expand $(\Omega - \vec{v} \cdot \vec{q})^{-2}$ which appears after integration by parts of (37). Thus, after finding the principal value of the integral, we obtain

$$\epsilon(\vec{q}, \Omega) \cong 1 - \frac{\Omega_{pe}^2 J_0^2(q_x \gamma_0)}{\Omega^2} \left[1 + \frac{3\langle(\vec{q} \cdot \vec{v})^2\rangle}{\Omega^2} + \frac{5\langle(\vec{q} \cdot \vec{v})^4\rangle}{\Omega^4}\right] \exp\left(-\frac{E_\gamma}{k_B T}\right). \tag{38}$$

By making q small in (38) and using (34) the modes of the collective excitation of the system are simply given by

$$\Omega_{\vec{q},\lambda} \cong \Omega_{pe} \exp\left(-\frac{E_\gamma}{2k_B T}\right) \left[J_0^2(q_x \gamma_0)\right]^{1/2}. \tag{39}$$

3. Results and discussion

Equation (39) is the main result we want to discuss. At small wave vectors the plasma energy is mainly determined by the electrons themselves. As q increases the terms for which $m \neq 0$ start to contribute and (38) is no longer valid because the second and third terms in the brackets contribute to a much complicated solution for the collective excitation frequency. We note that for the field-amplitude case ($m = 0$) the collective excitation frequency given by equation (39) is strongly dependent upon the frequency and amplitude through the parameters $E_\gamma = e^2 E^2 / 4 m_e \omega^2$ and $\gamma_0 = eE/m_e \omega^2$. Also, the effect of the electron-radiation field coupling is to lower the plasmon frequency by means of the factor $\exp(-E_\gamma / 2k_B T)$.

Proceeding further, in order to analyze the behavior of (39), we performed some numerical computations of collective mode frequencies for some typical plasmas, namely hot and diffuse plasma, gas discharges, warm plasma and hot plasma (see table 1). Since the external field confers a preferred direction (the x-axis) we postulated in all numerical computations that the plasmons are propagating along the x-direction ($q_x = q$ and $q_z = q_y = 0$). Firstly, we applied (39) for a gas discharge plasma and for a hot plasma. We assumed EM radiation frequencies ranging from $\omega = 3 \times 10^7$ s^{-1} to $\omega = \Omega_{pe}$, the plasma frequency. The low frequencies were included in the numerical simulations in order to seek attention to the fact that for field frequencies below the plasma frequency the electromagnetic wave in the plasma is strongly attenuated, even taking into account the small skin depth (δ) effect in such dense plasmas which is of the order of 0.01 mm.

Figure 1 shows the dispersion of the plasma mode frequency as a function of q for the above-considered plasmas. We have used $E = 10$ V m^{-1} and observed that there is no difference in such dispersions as expected. Secondly, we have applied (39) for a Gas Discharge and obtained the curves depicted in figure 2 which shows the plasma mode frequency as a function of EM frequency (ω) and intensity (E). We can see from this three-dimensional

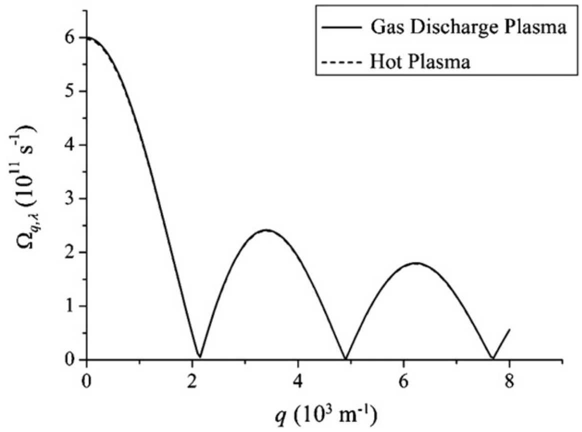

Figure 1. Dispersion of collective mode for a gas discharge plasma (solid line) and a hot plasma (dashed line) as a function of wavenumber, q, for an electric field amplitude $E = 10$ V m^{-1}. The EM frequency is $\omega = 2 \times 10^9$ s^{-1}.

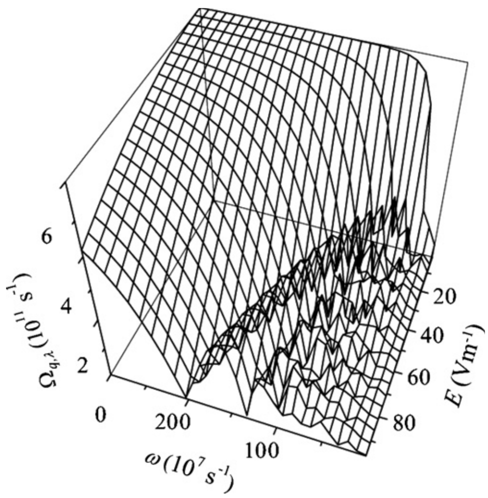

Figure 2. Collective mode frequency as a function of radiation frequency, ω, and radiation-field amplitude, E, for a gas discharge plasma.

plot that the effect of wave attenuation for low radiation frequency values (below the plasma frequency) is strongly reflecting the effect on the drastic attenuation of the collective modes as expected. For a hot plasma the collective mode frequency has the same behavior as that shown in figure 2, here the phase velocity of the excitations $v \approx v_{\text{th}}$ (thermal velocity). Also, we observe from figure 2 that the collective mode frequencies decrease for high radiation-field amplitudes. Thirdly, we have applied (39) for a gas discharge for three EM frequencies and plotted figure 3 from the results. In this figure, we observe that the plasma mode frequency has the same behavior as that observed in figure 2. Figure 4 shows the plasma mode frequency, (39), for a hot plasma, a gas discharge plasma and a warm plasma as a function of wavenumber (as appearing in the argument of the Bessel function).

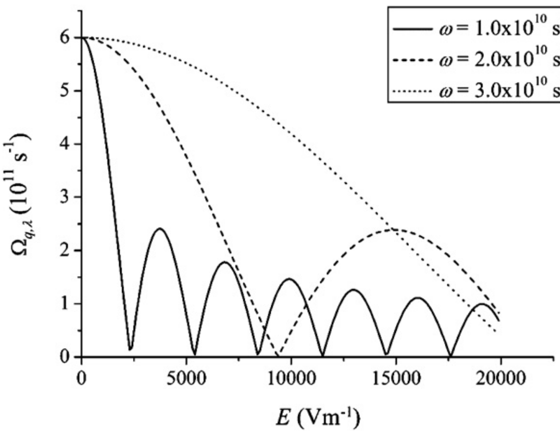

Figure 3. Collective mode frequency as a function of electric field amplitude, E, for a gas discharge for three EM frequencies, ω: $1 \times 10^{10}\ \mathrm{s}^{-1}$ (solid line), $2 \times 10^{10}\ \mathrm{s}^{-1}$ (dashed line) and $3 \times 10^{10}\ \mathrm{s}^{-1}$ (dotted line).

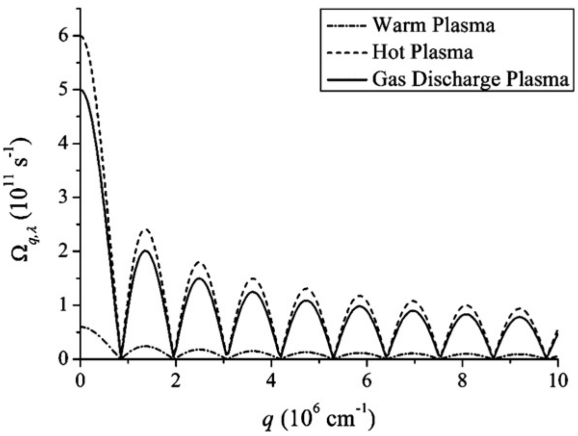

Figure 4. Collective mode frequency, (39), as a function of wavenumber (argument of the Bessel function) for a warm plasma (dot-dashed line), a hot plasma (dashed line) and a gas discharge plasma (solid line) for an EM frequency of $\omega = 3 \times 10^{9}\ \mathrm{s}^{-1}$.

Let us now associate our above finding with the high-frequency electrical conductivity in our nonmagnetized plasma. This is accomplished by taking account of the standard relation between the plasma conductivity and dielectric response functions [20], namely

$$\epsilon = 1 + \frac{4\pi}{\mathrm{i}\Omega}\sigma. \tag{40}$$

Hence, by making use of result (38) for small q and (40) we obtain the reactive plasma electrical conductivity (imaginary part) under a radiation field

$$\sigma_{\mathrm{reac}}(\Omega) = \frac{\Omega_{\mathrm{pe}}^2 \exp\left(-\frac{E_\gamma}{k_{\mathrm{B}}T}\right) J_0^2(q_x\gamma_0)}{4\pi\Omega}. \tag{41}$$

It then follows that the reduction of plasmon frequency under a radiation field through (39) can also be understood as the suppression of the reactive plasma electrical conductivity under a radiation field. That is, by increasing the field parameter E_γ in (41) which involves the field amplitude and frequency, respectively, σ_{reac} decreases because of the exponential factor thereby making the plasma electrons less mobile. On the other hand, since the field-amplitude process is proportional to the square of the Bessel function $J_0(\gamma_0 q_x)$ we see that for a given q_x (q is small but not zero) and a given γ, the Bessel function of zeroth order oscillates with the field amplitude and as a consequence we have oscillations of the reactive conductivity. Therefore, the effect of the Bessel function factor in (41) is to modulate the reactive conductivity as a function of radiation-field amplitude. We then have two effects, namely the suppression of reactive plasma conductivity as the field amplitude increases and its modulation through the Bessel function.

At this stage it is worthwhile to mention the effect of the radiation field on classical Landau damping. In the absence of collisions, i.e., for a collisionless plasma for which $\Omega_{\text{pe}}t \gg 1$ (t is the electron relaxation time), the plasma waves are Landau damped. However, in the presence of a radiation field, the external field provides a drift velocity to the electrons such that whenever the drift velocity is greater than the phase velocity of the plasma wave, there is a transference of energy from the electrons to the plasma waves at the expense of EM field and the plasmon population grows. If this growth rate in the radiation field becomes greater than the Landau damping coefficient we have plasma wave amplification.

In order to test the predictions put forward in the current paper, we suggest a numerical simulation. Since the formalism used here is similar to solid state plasmas, we propose a simulation analogous to that of [21] where the dynamics of the degenerate electron gas of a thin Cu metal film subject to an intense and ultra-short UV electromagnetic pulse in resonance with the electron plasma frequency of the system was investigated. In such a numerical simulation however the electron distribution function of the thin Cu metal film has to be considered a non degenerate electron distribution to fit our above results using the Maxwellian distribution function for electrons. The results of the simulation in [21] show that a collective excitation of electrons by resonance absorption leads to the formation of plasma waves in the metal film. The same behavior is expected for the simulation by using the present Maxwellian distribution function for electrons since the characteristic energy of plasma electrons—in our particular case—is independent of electron distribution function.

Moreover, the reduction of the plasmon frequency under a radiation field can be associated with the suppression of the plasma electrical conductivity under a radiation field. Even though quantum effects are negligible in our plasma system the results put forward above are consistent with the experimental observation of a suppressed conductivity in a quasi-two-dimensional electron gas under a THz radiation field [22–24] where quantum effects are important.

4. Conclusion

In the present paper, we made a theoretical study of the dielectric function of a radiation-driven electron–gas plasma. In order to find the time-dependent wavefunction for electrons under the radiation field we made use of unitary transformations of quantum mechanics. The charge fluctuation of the system was then evaluated by using the time-dependent perturbation technique in order to obtain an expression for a local potential from which an expression for the dielectric function was derived. And then the classical limit was taken in order to apply the results to macroscopic—i.e. classical—plasmas, as successfully accomplished in a number of cases [11, 13, 14]. The spectrum of the collective excitation was then calculated and shows strong dependence upon the intensity and frequency of radiation field. As a consequence of this effect one observes the suppression of the reactive conductivity.

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